Yellow Sheets

Functions and Inverses

by Jason C. McDonald

About Yellow Sheets

No, they're not yellow (unless you printed them on that color paper.) Back when I was taking Pre-Calculus, I had a five subject notebook I used for my class notes. I would write especially important facts on the yellow divider pages, so I could find them easily later. Since then, I have frequently referenced those “yellow sheets” while tutoring at our local community college, often copying them down for the tutee for keep. Finally, I decided to create a high-quality set of these “yellow sheets”, modeled after the charts I have successfully used in tutoring.

Thus, “Yellow Sheets” refers to the theory of content: these one-page charts and graphs contain only that information which you would write on a notebook divider page in your class notes.

Using Yellow Sheets

These are intended to be learning *tools*. They are no substitution for one-on-one explanations, lectures, reading the textbook, or doing the work. Tutors using Yellow Sheets should consider working the example problem with the student, explaining all the concepts contained therein.

About Jason C. McDonald

Jason C. McDonald is the CEO and Lead Developer of MousePaw Games, which is dedicated to furthering education through technology, as well as through resources such as this.

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Functions

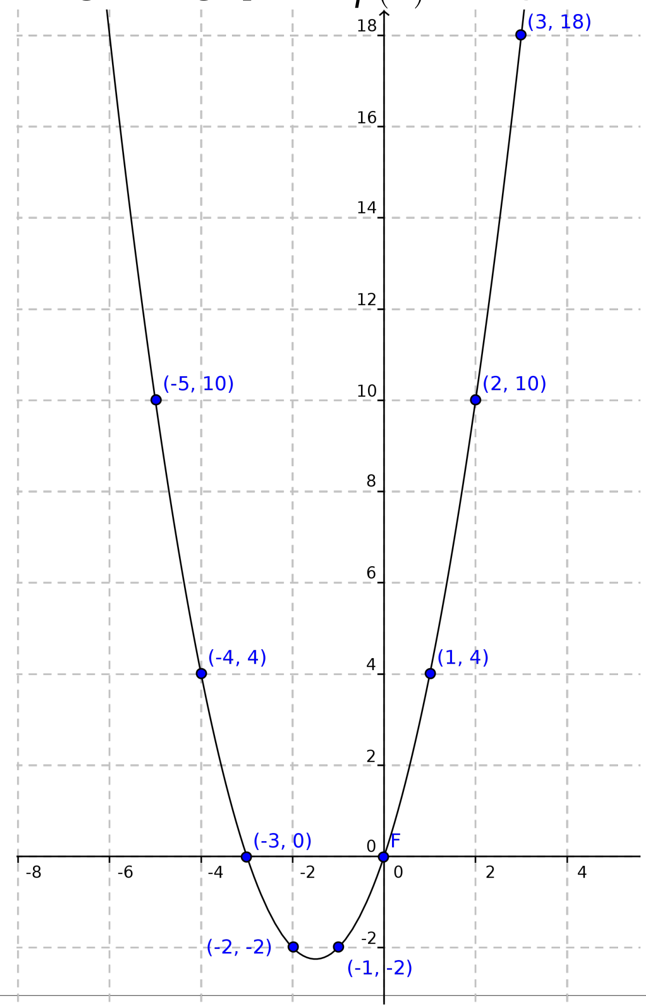
A **function** allows you to input a value into one variable (usually x) and get another value. This output is usually called $f(x)$ or y .

$$f(x)=x^2+3x$$
$$f(2)=(2)^2+3(2)$$
$$=4+6$$
$$=10$$
$$\dots$$
$$f(2)=10$$

A function can also be graphed. The graph of a function consists of every x value and corresponding y or $f(x)$ value(s).

X	Y		X	Y		X	Y
-5	10		-1	-2		3	18
-4	4		0	0		4	28
-3	0		1	4		5	40
-2	-2		2	10			

The following is the graph of $f(x)=x^2+3x$



Combining Functions

What happens if we need to combine the output of two functions? We can **combine** the functions.

$$f(x)+g(x)$$
$$f(x)-g(x)$$
$$f(x)*g(x)$$
$$\frac{f(x)}{g(x)}$$

Example 1: Two functions, no value for x.

$$f(x)=2x, g(x)=x^2+3x+7$$
$$f(x)*g(x)$$

Step 1: Write both functions in the combination.

$$(2x)*(x^2+3x+7)$$

Step 2: If possible, simplify. In this example, distribute.

$$(2x^2+6x+14)$$

NOTE: You won't always be able to simplify.

Example 2: Two functions, given value for x.

$$f(x)=\sqrt{x+2}, g(x)=x+9, x=4$$
$$g(4)-f(4)$$

Step 1: Write both functions in the combination.

$$x+9-\sqrt{x+2}$$

Step 2: Sometimes, it will be easier to simplify now. In this case, we cannot simplify the combination.

$$x+9-\sqrt{x+2}$$

Step 3: Place the given x value into the combination.

$$(4)+9-\sqrt{(4)+2}$$

Step 4: Simplify.

$$13-\sqrt{6}$$

NOTE: Take note of your domain. Sometimes, combinations will resolve domain issues, and sometimes it will create them. Here, the new domain is $x \neq 3$.

$$f(x)=x-3, g(x)=x^2+3x+7$$
$$g(x) \div f(x)$$
$$\frac{x^2+3x+7}{x-3}$$

Composing Functions

Sometimes we want to perform multiple functions on a number, like this... $f(x)=x^2+3x$ $g(x)=x-7x$ $x=2$ ... $g(2)=-12$ $f(10)=108$ We can make this easier by creating a new function that combines both $f(x)$ and $g(x)$. This is called **composition.** $f\circ g(x)$ or $f(g(x))$ To find “f of g of x”, we need to recognize which function we use first (the $g(x)$). That is the “inside” function. The “inside” function goes inside of the “outside” function $f(x)$, which is the second function we use. **Step 1:** Write the outside function, $f(x)$, with open parenthesis in place of every x. $f\circ g(x)=(\quad)^2+3(\quad)$	**Step 2:** Write the inside function, $g(x)$, inside each set of parenthesis. $f\circ g(x)=(x-7x)^2+3(x-7x)$ **Step 3:** Simplify. $f\circ g(x)=36x^2-18x$ Now we can put in our initial x, and we will get the same final answer as we did earlier. $f\circ g(2)=36(2)^2-18(2)$ $=36(4)-36$ $=144-36$ $=108$ We can nest any function inside of any other function by the same process. For example... $g\circ g(x)=(\quad)-7(\quad)$ $g\circ g(x)=(x-7x)-7(x-7x)$ $g\circ g(x)=x-7x-7x+49x$ $g\circ g(x)=50x-14x$ $g\circ g(x)=36x$

Inverse Functions

Wouldn't it be great to have an undo button in math? There is one: **inverse functions.** Let's find an inverse for...

$$f(x)=3x+18$$

Step 1: Write f(x) as y.

$$y=3x+18$$

Step 2: Swap all x and y.

$$x=3y+18$$

Step 3: Solve for y. First, get all y terms on one side, and all other terms on the other.

$$x-18=3y$$

$$3y=x-18$$

Step 4: Try to get down to a single y, if you have more than one.

$$\frac{3y}{3}=\frac{x-18}{3}$$

$$y=\frac{x-18}{3}$$

Step 5: Rewrite y as $f^{-1}(x)$ (f inverse of x).

$$f^{-1}(x)=\frac{x-18}{3}$$

Step 6: Check by composing $f(x)$ and $f^{-1}(x)$ (it doesn't matter which is inside and which is outside). The result should always be just x.

$$f\circ f^{-1}(x)=3\left(\frac{x-18}{3}\right)+18$$

$$f\circ f^{-1}(x)=x-18+18$$

$$f\circ f^{-1}(x)=x$$

WARNING: Always do the math to find the inverse and to check! Some functions can be tricky. There are a few functions which are actually their own inverses, though you'd never guess it.

The Point of Inverses

It is no coincidence that you swap x and y in step 2. If you look at an XY chart of a function and its inverse, the X and Y actually do swap.

$f(x)=3x+18$		$f^{-1}(x)=\frac{x-18}{3}$	
X	Y	X	Y
5	33	33	5
10	48	48	10

The domains and ranges also swap, which is handy in finding the range of a function...just find the domain of its inverse!