

Yellow Sheets

Factoring and Friends

by Jason C. McDonald

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About Yellow Sheets

No, they're not yellow (unless you printed them on that color paper.) Back when I was taking Pre-Calculus, I had a five subject notebook I used for my class notes. I would write especially important facts on the yellow divider pages, so I could find them easily later. Since then, I have frequently referenced those “yellow sheets” while tutoring at our local community college, often copying them down for the tutee for keep. Finally, I decided to create a high-quality set of these “yellow sheets”, modeled after the charts I have successfully used in tutoring.

Thus, “Yellow Sheets” refers to the theory of content: these one-page charts and graphs contain only that information which you would write on a notebook divider page in your class notes.

Using Yellow Sheets

These are intended to be learning *tools*. They are no substitution for one-on-one explanations, lectures, reading the textbook, or doing the work. Tutors using Yellow Sheets should consider working the example problem with the student, explaining all the concepts contained therein.

About Jason C. McDonald

Jason C. McDonald is the CEO and Lead Developer of MousePaw Games, which is dedicated to furthering education through technology, as well as through resources such as this.

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Factoring Method 1: Magic Numbers

NOTE: Not all methods of factoring are covered in these yellow sheets.

NOTE: This method works best with quadratic equations.

$$2x^2 + 4x - 16$$

Step 1: Set up the parenthesis and signs. Think about FOILING: the last terms of the factors will add to be a positive number, and multiply to be a negative number. Put the leading coefficient and an x on the left side of each factor.

$$(? + ?)(? - ?)$$

Step 2: Put the leading coefficient and an x as the left side of each factor.

$$(2x + ?)(2x - ?)$$

Step 3: Multiply the leading coefficient by the constant. Divide that number by the “magic numbers” to determine it's prime factors. (You might need to look at the other prime numbers too.)

$$2 \cdot 16 = 32$$

Magic Numbers: 2, 3, 5, 7, 11

$$32 \div 2 = 16$$

$$32 \div 3 \approx 10.66$$

$$16 \div 5 = 6.4$$

$$16 \div 7 \approx 4.57$$

$$16 \div 11 \approx 2.91$$

Step 4: From Step 3, we can eliminate all multiples of 3, 5, 7, and 11 as factors. Let's look at multiples of 2 as factors.

$$32 \div 2 = 16, \quad 32 \div 4 = 8, \quad 32 \div 32 = 1$$

$$1 \cdot 32 = 32, \quad 2 \cdot 16 = 32, \quad 4 \cdot 8 = 32$$

Step 5: Figure out which pair above adds up to the middle term (4) Remember from Step 1 that one number is positive, and one is negative.

$$8 - 4 = 4$$

Step 6: Place the pair in the parenthesis and simplify. You may need to try a couple of arrangements.

$$(2x + 8)(2x - 4)$$

$$(x + 4)(2x - 4) = 2x^2 + 4x - 16$$

Special Case: Difference of Squares

When you see an x^2 minus a perfect square, you have a **difference of squares**. It works as follows...

$$(x^2 - 9) = (x + 3)(x - 3)$$

$$(x^2 - 25) = (x + 5)(x - 5)$$

What are the Magic Numbers? These are the five most common prime numbers. Eliminating one as a factor eliminates all of its multiples. You may need to look at other prime numbers, though this is rare.

Factoring Method 2: Quadratic Formula

NOTE: Not all methods of factoring are covered in these yellow sheets.

NOTE: This method only works with quadratic polynomials.

$$2x^2 + 4x - 6$$

The Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

It's just one of those things you have to memorize.

Step 1: Determine your a, b, and c. These come from your polynomial as follows...

$$ax^2 + bx + c$$

...

$$2x^2 + 4x - 6$$

Step 2: Plug your a, b, and c into the quadratic formula.

$$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(2)(-6)}}{2(2)}$$

Step 3: Simplify.

$$x = \frac{-4 \pm \sqrt{16 - 8(-6)}}{4}$$

$$x = \frac{-4 \pm \sqrt{16 + 48}}{4}$$

$$x = \frac{-4 \pm \sqrt{64}}{4}$$

$$x = \frac{-4 \pm 8}{4}$$

$$x = \frac{4(-1 \pm 2)}{4}$$

$$x = -1 \pm 2$$

$$x = 1, -3 \text{ or } (x-1)(x+3)$$

Step 4: If you're factoring, FOIL your factors. You may need a constant – usually the leading coefficient of the original polynomial.

$$(x-1)(x+3) = x^2 + 2x - 3$$

$$2(x^2 + 2x - 3) = 2x^2 + 4x - 6$$

$$2(x-1)(x+3) = 2x^2 + 4x - 6$$

Factoring Method 3: Long Division

NOTE: Not all methods of factoring are covered in these yellow sheets.

NOTE: This method works well with higher degree polynomials, but requires one known factor.

$$x^3 + 4x^2 - 11x - 30; (x+5) \text{ is a factor}$$

Step 1: Lay out the long division problem, with the factor as divisor, and the polynomial as dividend.

$$x+5 \overline{) x^3 + 4x^2 - 11x - 30}$$

Step 2: Divide the first term of the dividend by the first term of the divisor.

$$x+5 \overline{) x^3 + 4x^2 - 11x - 30}$$

$$\phantom{x+5 \overline{) }} \underline{x^3}$$

Step 3: Multiple the first term of the dividend by the new quotient term.

$$x+5 \overline{) x^3 + 4x^2 - 11x - 30}$$

$$\phantom{x+5 \overline{) }} \underline{x^3 + 5x^2}$$

Step 4: Subtract, like in normal division, bringing down the other factors from the dividend.

$$x+5 \overline{) x^3 + 4x^2 - 11x - 30}$$

$$\phantom{x+5 \overline{) }} \underline{- x^3 + 5x^2}$$

$$\phantom{x+5 \overline{) }} \underline{- x^2 - 11x - 30}$$

Step 5: Continue like this, dividing the divisor into the result.

$$x+5 \overline{) x^3 + 4x^2 - 11x - 30}$$

$$\phantom{x+5 \overline{) }} \underline{x^3 + 5x^2}$$

$$\phantom{x+5 \overline{) }} \underline{- x^2 - 11x - 30}$$

$$\phantom{x+5 \overline{) }} \underline{- x^2 - 5x}$$

$$\phantom{x+5 \overline{) }} \underline{- 6x - 30}$$

$$\phantom{x+5 \overline{) }} \underline{0}$$

Step 6: You may need to factor the quotient, otherwise it is the other factor of the dividend.

$$x^2 - x - 6 = (x+2)(x-3)$$

$$(x+2)(x-3)(x+5) = x^3 + 4x^2 - 11x - 30$$

Factoring Method 4: Synthetic Divison

NOTE: Not all methods of factoring are covered in these yellow sheets.

NOTE: This method works well with higher degree polynomials, but requires one known factor.

$$x^3 - 6x + 9; (x+3) \text{ is a factor}$$

Step 1: Rewrite the polynomial with any missing factors, if any.

$$x^3 + 0x^2 - 6x + 9$$

Step 2: Write the coefficients of the polynomial ONLY. Inside the left box, write the zero of the known factor.

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -6 & 9 \end{array}$$

Step 3: Bring the first coefficient down directly.

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -6 & 9 \\ \downarrow & & & & \\ \hline & 1 & & & \end{array}$$

Step 4: Multiply this number by the one in the left box (the zero), and put the result below the second coefficient.

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -6 & 9 \\ \downarrow -3 & & & & \\ \hline & 1 & & & \end{array}$$

Step 5: Add.

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -6 & 9 \\ + \downarrow -3 & & & & \\ \hline & 1 & -3 & & \end{array}$$

Step 6: Multiply the new number by the zero, and place below the next coefficient.

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -6 & 9 \\ + \downarrow -3 & & & & \\ \hline & 1 & -3 & & \end{array}$$

Step 7: Subtract this number from the coefficient.

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -6 & 9 \\ + \downarrow -3 & & & & \\ \hline & 1 & -3 & 3 & \end{array}$$

Step 8: Repeat until the end. The last number goes in the lower right box. That is your remainder.

$$\begin{array}{r|rrrr} -3 & 1 & 0 & -6 & 9 \\ + \downarrow -3 & & & & \\ \hline & 1 & -3 & 3 & 0 \end{array}$$

Step 9: If you get zero as a remainder, then the numbers at the bottom are the coefficients of a factor polynomial. If you don't get zero as a remainder, then the zero did NOT come from a factor.

$$(1x^2 - 3x + 3)(x + 3) = x^3 - 6x + 9$$

NOTE: $x^2 - 3x + 3$ has no rational factors.

Factoring Method 5: Completing the Square

NOTE: Not all methods of factoring are covered in these yellow sheets.

Some quadratic polynomials can be rearranged through a process called “completing the square” in order to factor or to solve for x quickly.

$$2x^2 + 12x + 10$$

Step 1: Set the polynomial equal to zero (unless it is already set equal to something).

$$2x^2 + 12x + 10 = 0$$

Step 2: Subtract the constant from both sides to move it.

$$2x^2 + 12x = -10$$

Step 3: If there is a leading coefficient, factor it out (of the left side.)

$$2(x^2 + 6x) = -10$$

Step 4: Look at the polynomial. Take half of the linear term's coefficient and square it.

$$6 \div 2 = 3$$

$$3^2 = 9$$

Step 5: Add that number to both sides. On the left, we put it inside the parenthesis. Since we really added 18 (9x2) to the left, we need to add 18 to the right as well.

$$2(x^2 + 6x + 9) = -10 + 18$$

Step 6: Factor the left polynomial. Notice the sole factor is half of the linear term coefficient. Simply the right.

$$2(x+3)^2 = 8$$

Step 7: Divide both sides by the number we factored out.

$$\frac{2(x+3)^2}{2} = \frac{8}{2}$$

$$(x+3)^2 = 4$$

Step 8: Take the square root of both sides. Remember the negative square root!

$$\sqrt{(x+3)^2} = \sqrt{4}$$

$$x+3 = \pm 2$$

Step 9: Solve for x.

$$x = \pm 2 - 3$$

$$x = -1, x = -5$$

$$(x+1)(x+5)$$

Step 10: If you're factoring, FOIL your factors. You may need a constant – usually the leading coefficient of the original polynomial.

$$(x+1)(x+5) = x^2 + 6x + 5$$

$$2(x^2 + 6x + 5) = 2x^2 + 12x + 10$$

$$2(x+1)(x+5) = 2x^2 + 12x + 10$$

Factoring with Imaginary Numbers

NOTE: Not all methods of factoring are covered in these yellow sheets.

So we've arrived at an unfactorable polynomial. Welcome to the world of imaginary, or "complex", numbers.

$$x^2 - 3x + 3$$

Step 1: Set the polynomial equal to zero.

$$x^2 - 3x + 3 = 0$$

Step 2: Complete the square.

$$\left(x + \frac{3}{2}\right)^2 = -\frac{3}{4}$$

Step 3: Take the square root of both sides.

$$\sqrt{\left(x + \frac{3}{2}\right)^2} = \pm \sqrt{-\frac{3}{4}}$$

Step 4: Notice that the right is irrational, because you cannot take the square root of a negative number. Break the right side into two square roots, with one of them being $\sqrt{-1}$.

$$\sqrt{\left(x + \frac{3}{2}\right)^2} = \pm \sqrt{\frac{3}{4}} * \sqrt{-1}$$

Step 5: Finish taking the square root, writing $\sqrt{-1}$ as i .

$$x + \frac{3}{2} = \pm \sqrt{\frac{3}{4}} * i$$

Step 6: Solve for x. The "real" part of the answer should come before the "imaginary" part.

$$x = -\frac{3}{2} \pm \sqrt{\frac{3}{4}} i$$

Step 7: Rearrange and rewrite as factors. Remember, complex factors always come in pairs!

$$\left(x + \frac{3}{2} + \sqrt{\frac{3}{4}} i\right) \left(x + \frac{3}{2} - \sqrt{\frac{3}{4}} i\right)$$

NOTE: In some cases, you may be asked to rewrite the factors to remove the fractional constants.

Special Case: Sum of Squares

When you see an x^2 plus a perfect square, you have a **sum of squares**, which is actually a pair of complex (imaginary) factors. It works as follows...

$$x^2 + 9 = (x \pm 3i)$$

$$x^2 + 16 = (x \pm 4i)$$

$$x^2 + 25 = (x \pm 5i)$$