Yellow Sheets

Exponentials and Logarithms

by Jason C. McDonald

About Yellow Sheets

No, they're not yellow (unless you printed them on that color paper.) Back when I was taking Pre-Calculus, I had a five subject notebook I used for my class notes. I would write especially important facts on the yellow divider pages, so I could find them easily later. Since then, I have frequently referenced those “yellow sheets” while tutoring at our local community college, often copying them down for the tutee for keep. Finally, I decided to create a high-quality set of these “yellow sheets”, modeled after the charts I have successfully used in tutoring.

Thus, “Yellow Sheets” refers to the theory of content: these one-page charts and graphs contain only that information which you would write on a notebook divider page in your class notes.

Using Yellow Sheets

These are intended to be learning *tools*. They are no substitution for one-on-one explanations, lectures, reading the textbook, or doing the work. Tutors using Yellow Sheets should consider working the example problem with the student, explaining all the concepts contained therein.

About Jason C. McDonald

Jason C. McDonald is the CEO and Lead Developer of MousePaw Games, which is dedicated to furthering education through technology, as well as through resources such as this.

Visit MousePaw Games online: www.mousepawgames.com
More about Jason C. McDonald: www.indeliblebluepen.com

About The Licensing

Yellow Sheets by Jason C. McDonald is licensed under a Creative Commons Attribution 4.0 International License. Basically, you're free to reproduce and distribute freely, as long as you give credit where credit is due. Be sure to read the full license details at http://creativecommons.org/licenses/by/4.0/.

The Three Exponent Equations

A typical exponential equation has three parts: **base**, **exponent**, and **result**. (We'll be using this color coding throughout.)

There is an equation for finding any of the three parts. Any equation can be written in all three formats.

Exponential Equation: Use me to find the **result**.

$$base^{exponent} = result \qquad 2^3 = 8$$

Root Equation: Use me to find the **base**.

$$\sqrt[exponent]{result} = base \qquad \sqrt[3]{8} = 2$$

Logarithmic Equation: Use me to find the **exponent**.

$$\log_{base} result = exponent \qquad \log_2 8 = 3$$

→**Where is the base?** For both exponential and logarithmic equations, remember the phrase "**base** is in the **bottom**."

Solving Exponential and Logarithmic Equations

Exponential Equations	Logarithmic Equations
If bases are the same, set exponents equal. $2^{2x}=2^4$ $2x=4$ $x=2$	If log = number, write as exponential and solve. $\log_3 x=3$ $3^3=x$ $27=x$ $x=27$
If bases are not the same, take the ln of both sides. $2^{2x}=3^4$ $\ln 2^{2x}=\ln 81$ $2x\ln 2=\ln 81$ $\frac{2x\ln 2}{2\ln 2}=\frac{\ln 81}{2\ln 2}$ $x=\frac{\ln 81}{2\ln 2}$	If log = log (same base), set results equal. You may need to simply first. $4\log_4 7x=\log_4 9072$ $\log_4 7x^4=\log_4 9072$ $x^4=1296$ $\sqrt[4]{x}=\sqrt[4]{1296}$ $x=6$

Properties of Logarithms

NOTE: All variables are real numbers. $a>0, a\neq 1, b>0, b\neq 1, M>0, \wedge N>0.$

$$\log_a 1=0$$
...
$$\log_{17} 1=x$$
$$17^x=1$$
$$17^0=1$$
$$x=0$$
...
$$\log_{17} 1=0$$

$$\log_a a=1$$
...
$$\log_{12} 12=x$$
$$12^x=12$$
$$12^1=12$$
$$x=1$$
...
$$\log_{12} 12=1$$

$$\log_a M^r=r\log_a M$$
...
$$\log_2 2^3=x\log_2 2$$
$$\log_2 8=x(1)$$
$$[2^3=8]$$
$$3=x$$
...
$$\log_2 2^3=3\log_2 2$$

$$a^{\log_a M}=M$$
...
$$3^{\log_3 27}=x$$
$$[\log_3 27=3]$$
$$3^3=x$$
$$27=x$$
...
$$3^{\log_3 27}=27$$

$$\log_a a^r=r$$
...
$$\log_4 4^{17}=x$$
$$17\log_4 4=x$$
$$17(1)=x$$
$$17=x$$
...
$$\log_4 4^{17}=17$$

$$a^x=e^{x\ln a}$$
...
$$2^3=e^{3\ln 2}$$
$$8=8$$

$$\log_a(M*N)=\log_a M+\log_a N$$
...
$$\log_2(4*8)=\log_2 4+\log_2 8$$
$$\log_2(32)=(2)+(3)$$
$$[2^5=32]$$
$$(5)=(5)$$

$$\log_a\left(\frac{M}{N}\right)=\log_a M-\log_a N$$
...
$$\log_2\left(\frac{8}{4}\right)=\log_2 8-\log_2 4$$
$$\log_2(2)=(3)-(2)$$
$$[2^2=1]$$
$$(1)=(1)$$

$$\log_a M=\log_a N \Leftrightarrow M=N$$
...
$$\log_2 4=\log_2 x$$
$$[2^2=4]$$
$$2=\log_2 x$$
$$2^2=x$$
$$4=x$$

Change Of Base Formula

Many times, you will not be able to solve a logarithm as it is written. You will need to change its base, whether to use the properties of logs to simplify and solve, or to enter it into the calculator.

We can change the base of a logarithm using the **change of base formula**.

$$a = old\ base$$
$$M = result$$
$$b = new\ base$$
$$\log_b M = \frac{\log_b M}{\log_b a}$$

REMEMBER: Base is in the bottom.

Calculators can only calculate common and natural logs. However, we can find the value of any logarithm on the calculator if we first use the change of base formula.

$$\log_3 81 = \frac{\ln 81}{\ln 3} = (calculator) = 4$$